

First excitations of the spin 1/2 Heisenberg antiferromagnet on the kagomé lattice

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Abstract. We study the exact low energy spectra of the spin 1/2 Heisenberg antiferromagnet on small samples of the *kagomé* lattice of up to $N = 36$ sites. In agreement with the conclusions of previous authors, we find that these low energy spectra contradict the hypothesis of Néel type long range order. Certainly, the ground state of this system is a spin liquid, but its properties are rather unusual. The magnetic ($\Delta S = 1$) excitations are separated from the ground state by a gap. However, this gap is filled with nonmagnetic ($\Delta S = 0$) excitations. In the thermodynamic limit the spectrum of these nonmagnetic excitations will presumably develop into a gapless continuum adjacent to the ground state. Surprisingly, the eigenstates of samples with an odd number of sites, *i.e.* samples with an unsaturated spin, exhibit symmetries which could support long range chiral order. We do not know if these states will be true thermodynamic states or only metastable ones. In any case, the low energy properties of the spin 1/2 Heisenberg antiferromagnet on the *kagomé* lattice clearly distinguish this system from either a short range RVB spin liquid or a standard chiral spin liquid. Presumably they are facets of a generically new state of frustrated two-dimensional quantum antiferromagnets.

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1 Introduction

Zeng and Elser [1] were the first to point out that the spin 1/2 Heisenberg antiferromagnet on a *kagomé* lattice (hereafter called KHA) could be disordered. Since then many studies have been devoted to this system. All quantum approaches based on either exact diagonalizations [2,3], on perturbational series expansions [4] or on high temperature expansions [5] point to a disordered ground state. Previous exact diagonalization studies [2,3] have shown that spin-spin, spin nematic, spin Peierls and chiral-chiral correlations decrease very rapidly with distance. Therefore the idea that the KHA is a spin liquid is by now widely accepted. The precise nature of the spin liquid state of the KHA and of its low lying excitations is, however far from clear. Some approaches point to a liquid of short range dimers [6–8], others suggest that the KHA might be a candidate for a chiral spin liquid state of the kind first proposed for the triangular antiferromagnet by Kalmeyer and Laughlin [9–11]. It seems to be the common view that in both pictures any excitations, magnetic or nonmagnetic ones, are separated from the ground state by a finite gap.

While for the magnetic excitations this view is supported by all previous numerical work on the KHA [1,3], the situation is less clear for the nonmagnetic excitations. The only piece of evidence for a finite albeit small gap separating the singlet excitations of the KHA from its ground state comes from the work of Zeng and Elser [8]. Elaborating on the idea of a short-range dimer liquid these authors construct an effective low energy Hamiltonian for the singlet subspace of the KHA, which as expected, exhibits a small gap above its ground state. The question of whether one of these pictures, that of a short range RVB spin liquid or that of a chiral spin liquid, applies to the KHA has led us to study not only the energies but also the symmetry properties of a very large number of low lying levels of the exact spectra of the KHA.

2 Numerical approach and some general results

Using the Lanczos technique and a complete group theoretical analysis (as in Ref. [12]) we computed the low

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energy spectra of the Heisenberg Hamiltonian

$$\mathcal{H} = J \sum_{\langle ij \rangle} 2\mathbf{S}_i \cdot \mathbf{S}_j \quad (1)$$

for small samples of the *kagomé* lattice on a 2-dimensional torus with periodic and twisted boundary conditions (in the following $J = 1$). We have obtained a very large number of low lying levels in each irreducible representation (IR) of $SU(2)$ and of the space groups of the $N = 9, 12, 15, 18, 21, 24, 27, 36$ samples.

Table 1. Lowest energies per bond associated with the four different \mathbf{k} -points in the Brillouin zone of the $N = 36$ sample.

\mathbf{k}	E/N
$(0, 0)$	-0.438377
$(\frac{2\pi}{3}, 0)$	-0.437851
$(0, \frac{2\pi}{\sqrt{3}})$	-0.437585
$(\frac{4\pi}{3}, 0)$	-0.438096

As in all spectra of Heisenberg antiferromagnets that we have examined, the ground state energy of the S -subspaces increases with the total spin S . However, the spectra of the finite samples of the KHA do not show the pattern that is characteristic for systems which exhibit Néel type long range order in the thermodynamic limit as, *e.g.* the triangular antiferromagnet (for a detailed discussion of the signature of long range order in the spectra of finite samples see Ref. [13]). Instead, all the signatures of a “liquid” are present in the low energy spectra of the KHA:

- i) The lowest levels associated with the different momenta \mathbf{k} in the Brillouin zones of the samples are almost independent of \mathbf{k} . For instance, for the largest sample ($N = 36$), the energies of these lowest levels vary by less than 0.2% when \mathbf{k} is varied through the four inequivalent \mathbf{k} -points of the Brillouin zone (see Tab. 1). This absence of dispersion excludes the possibility of a broken translational symmetry.
- ii) The system is extremely soft against any twist of the boundary conditions [14].
- iii) The size dependence of the ground state energy is more than one order of magnitude smaller than for the triangular antiferromagnet in the same range $9 \leq N \leq 36$.

3 Magnetic gap and nonmagnetic excitations

Our results, (Fig. 1), confirm the conclusion of Elser and coworkers [1,3] about the existence of a gap for the magnetic $\Delta S = 1$ excitations. The thermodynamic limit of this gap cannot be extracted from the present data with high accuracy. The extrapolations shown in Figure 1 point to a lower bound of about $J/20$ (see also the discussion in Ref. [14]). In any case, the value of the magnetic gap

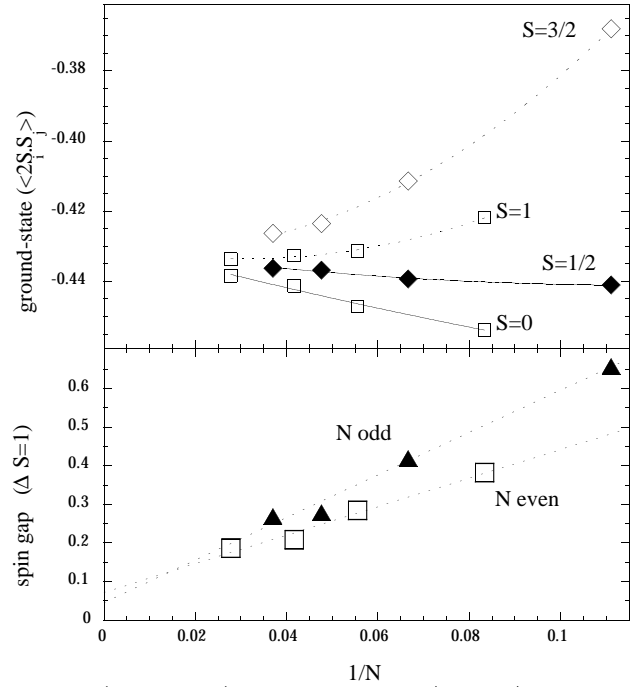


Fig. 1. (upper part) Energy per bond $\langle 2\mathbf{S}_i \cdot \mathbf{S}_j \rangle$ in the ground states of S_{min} and $S_{min} + 1$. The energy per bond in the thermodynamic limit is approximately $2\langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle = -0.43$. (lower part) Spin gap *versus* $1/N$ for the even N and the odd N samples.

appears to be at least one order of magnitude smaller than the exchange energy J which is needed to break an isolated singlet pair. This is a first indication that the picture of the *kagomé* antiferromagnet as a spin liquid consisting of short-range singlet dimer pairs may be inappropriate. Nevertheless, it comes as a surprise that quite contrary to the standard picture of a spin liquid the magnetic gap of the KHA is filled with nonmagnetic excitations. This is illustrated in Figure 2, where we display the integrated density of states of the $N = 36$ sample. In this sample there are 183 singlet levels below the lowest triplet. Similarly, in the $N = 27$ the $\Delta S = 1$ gap is filled with 153 nonmagnetic excitations. For the samples we have examined, the number of nonmagnetic ($\Delta S = 0$) excitations within the magnetic gap grows roughly as α^N with the system size, where $\alpha \simeq 1.15$ and $\alpha \simeq 1.18$ for the even and odd samples, respectively. These results call for a few comments:

- i) The density of nonmagnetic levels above the ground states of the two largest samples ($N = 27, 36$) strongly suggests that in the thermodynamic limit the nonmagnetic excitation spectrum of the KHA is a *gapless continuum adjacent to the ground state*. Interestingly, an algebraic decay of certain correlation functions would be compatible with such a gapless nonmagnetic continuum. A candidate is the dimer-dimer correlation function,

$$C_{(i,j)\langle k,l \rangle} = \langle (\mathbf{S}_i \cdot \mathbf{S}_j)(\mathbf{S}_k \cdot \mathbf{S}_l) \rangle - \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle^2, \quad (2)$$

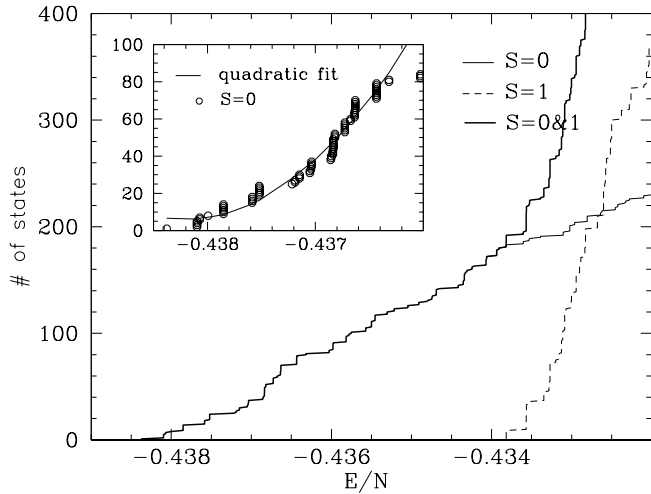


Fig. 2. Integrated density of states of the low lying levels of the $N = 36$ sample plotted *versus* the energy per bond (heavy line). The light and the broken line display the same quantity for the $S = 0$ and $S = 1$ subspaces separately. The inset shows a quadratic fit to the lowest 70 states of the spectrum.

which might decrease algebraically with the distance between the nearest neighbor bonds $\langle i, j \rangle$ and $\langle k, l \rangle$. Leung and Elser [3] have calculated this function for the ground state of the $N = 36$ sample of the KHA. Their result, a damped oscillatory decrease of $C_{\langle i, j \rangle \langle k, l \rangle}$, is not inconsistent with an algebraic decay. An algebraic decay of the dimer-dimer correlations would imply that the system is critical with respect to nonmagnetic quantum fluctuations. Whether this is the case for the KHA or not will be impossible to decide from finite size studies of the correlation function alone.

ii) The exponentially large number of low lying singlets of the spin 1/2 KHA is reminiscent of the ground state degeneracy of the corresponding classical model. Stability considerations show that only planar spin configurations qualify as true classical ground states [7, 15–17]. Their number and hence the ground state degeneracy of the classical KHA grows as 1.134^N with the system size N [18]. The different planar configurations are connected with each other by local rotations in spin space [16]. In a semiclassical picture these local rotations provide tunneling paths between the different planar configurations. It is therefore tempting to think of the low lying singlets of the quantum KHA as of tunnel-split classical ground states. This semiclassical picture has been pursued by von Delft and Henley [19]. As one of their main results these authors find that for spin 1/2 all the tunneling events involving the coherent rotation of the spins on simple loops on the *kagomé* lattice yield zero tunneling amplitudes due to destructive interferences between different tunneling paths. The smallest effective tunneling event requires the coherent rotation of the spins on two nested hexagons on the *kagomé* lattice involving 24 spins in total. Certainly, this scenario cannot explain the abundance of low lying sin-

glets in a sample consisting of only 21, 27, or 36 spins. In our view, this high density of low lying singlets in small samples of the KHA is a strong argument against the validity of the semiclassical approach for the spin 1/2 KHA.

iii) Another comparison that suggests itself is between the density of low lying singlets and the dimension of the nearest neighbor valence-bond basis on the *kagomé* lattice which grows as $2^{N/3} = 1.26^N$ with the system size [6]. From this point of view the variational approach of Zeng and Elser [8] which builds on the valence bond basis seems fully justified. The main difference between the approximate singlet spectrum of Zeng and Elser [8] and our numerically exact result lies in the density of levels at the bottom of this spectrum (see Fig. 2). The absence of a gap above the ground state and the high density of very low lying levels in the exact spectrum suggests that a dimer product representation of the corresponding eigenstates will necessarily contain long range singlet pairs. The importance of such longer range singlets in the spin liquid picture of the KHA might have been anticipated from the work of Zeng and Elser who observed that the inclusion of first *and* second-neighbor singlet pairs in their variational Hilbert space led to a considerable improvement over the results obtained in a pure first-neighbor dimer basis.

iv) The exponential number of non magnetic states in the gap should be visible in various experimental situations: it may explain the vanishing of the neutron elastic forward scattering cross section [20] and the very weak field dependence of the low temperature heat capacity of SrCrGaO observed by Ramirez *et al.* [21]. Specific heat data and neutron scattering experiments both point to a density of states that increases linearly with the energy. In the very low energy region ($E/N = -0.4384 \dots -0.4364$) our numerical data are consistent with such a linear energy dependence of the density of states (see the inset in Fig. 2, where the integrated density of states is plotted); they are definitely not consistent with an energy independent density of states as it would be obtained for an ordinary spin glass [22].

4 Spin 1/2 excitations

4.1 Symmetries of the low lying $S=1/2$ eigenstates

The second surprise lies in the degeneracies of the low lying levels of the spectra of the samples with an odd number of spins ($N = 9, 21, 27$) and in the symmetries of the eigenstates associated with them. We are focusing here on the exponentially large number of spin doublets which fill the gap between the ground state and the lowest $S = 3/2$ eigenstate. On account of the finite size scaling of the available spectra, we can ascertain that in the thermodynamic limit all these eigenstates are complex and exhibit an at least twofold degeneracy in addition to their magnetic degeneracy. (In the following we shall consistently ignore the trivial spin degeneracy.) Hence all these states may participate in a spontaneous mechanism breaking the time reversal (and parity) invariance of the Heisenberg Hamiltonian. In the spectra of the smallest samples, there are

still some low lying levels which belong to 1D IRs of the space group and are hence nondegenerate, but when the size of the sample increases these levels are pushed towards higher energies: in the $N = 27$ sample there are 153 $S = 1/2$ levels below the lowest $S = 3/2$ level, only 3 of them belong to 1D IRs of the space group. The finite size extrapolation predicts that for odd sizes $N > 50$ no states from the 1D IRs of the space group will be left in the magnetic gap. This is indeed a very specific property of the KHA which we have never observed in the spectrum of any spin Hamiltonian on the triangular lattice: in the usual triangular Néel antiferromagnet the homogeneous low lying states belong to the trivial representation of the C_3 group, whereas in the present case the $S = 1/2$ levels of the trivial representations are pushed high up in the spectra.¹ This observation leads us to concentrate on possible new properties of this low lying $S = 1/2$ continuum.

4.2 Chiral observable and correlation function

All these low lying levels (except the 3 belonging to 1D IRs) can sustain non-zero expectation values of the chirality operator

$$\Xi_{123} = 2 \mathbf{S}_1 \cdot (\mathbf{S}_2 \times \mathbf{S}_3) = \text{Im}(P_{123}) \quad (3)$$

where P_{123} is the cyclic permutation of three spins around a triangular plaquette.

We have measured this quantity in the lowest lying states of the $N = 9, 21$ and 27 samples for triangles with side lengths $1, \sqrt{3}, 2, \sqrt{7}, 3$ (see Tab. 2). Compared with the eigenvalues $\pm\sqrt{3}/2$ of the operator Ξ_{123} in the $S = 1/2$ bound states of three spins, the measured expectation values are indeed quite small, and there is a sharp decrease from the $N = 9$ to the $N = 21$ sample. But in view of the data for $N = 21$ and $N = 27$ it is unclear how this observable will behave for larger sizes. The chiral-chiral correlation function (last lines of Tab. 2) is indeed short ranged, which may explain the quasi-absence of size effects between $N = 21$ and $N = 27$, and is indeed characteristic of a liquid.

In order to have an indication of the possible order of magnitude of the chirality in a pure chiral spin liquid, we have computed the expectation value of Ξ_{123} for the spin $1/2$ excitations of the Laughlin wave function [11] on the same small lattices; these expectation values are denoted (L.w-f) in Table 2. While for triangles of size 1 the L.w-f values are an order of magnitude larger than the exact results, they are of the same order of magnitude for larger sizes. By this comparison we do not want to suggest that the physics of the KHA is the same as that of the Laughlin picture of a chiral spin liquid (as has been found in Sect. 4.

¹ In the case of the $q = 0$ Néel order, enforced on the *kagomé* lattice by a large enough second neighbor antiferromagnetic coupling [23], the ground state of the odd samples does in fact belong to the two complex conjugate IRs of C_3 [23]. However in this case it is the only low lying state with this symmetry, all the others have much higher energies.

Table 2. Expectation values of the chiral operator for triangular loops of various sizes: d is the side-length of the triangle in units of the nearest neighbor distance. These expectation values have been computed in the chiral ground states of the odd samples (they are homogeneous $\mathbf{k} = (0, 0)$ states and belong to the two degenerate complex conjugate IRs of C_3). The lines with the entry (L.w-f) in the left column contain the expectation values of the same operators in the variational Laughlin wave-function of reference [11]. The last six lines contain the correlations between the chiralities on elementary triangular plaquettes for various distances.

	$\langle \text{Im}(P_{123}) \rangle$		
N	9	21	27
$d = 1$	0.2570	0.0353	0.0567
(L.w-f)	0.2780	0.4022	0.4033
$d = \sqrt{3}$	0.0613	0.0261	0.0241
(L.w-f)	0.1176	0.0105	0.0081
$d = 2$	0.0000	0.0094	0.0004
(L.w-f)	0.0000	0.0098	0.0071
$d = \sqrt{7}$		0.0013	0.0013
(L.w-f)		0.0035	0.0020
$d = 3$			0.0080
(L.w-f)			0.0015
$d = \sqrt{12}$			0.0000

	$\langle \text{Im}(P_{123})\text{Im}(P_{456}) \rangle - \langle \text{Im}(P_{123}) \rangle^2$	
shell-shell distance	$N = 21$	$N = 27$
0-0	0.7013	0.6990
0-1	-0.0012	0.0135
0-2	-0.0166	0.0131
0-3	-0.0017	0.0019
0-4	0.0072	-0.0000
0-5		-0.0029

above, the first excitations are different). We only want to point out that the exact results may be significant in spite of their numerically small values.

On the basis of results obtained by applying the $SU(2)$ Schwinger-boson approximation to samples of the KHA with sizes of up to $N = 72$ sites Sachdev argues that the chirality of the ground state goes to zero with increasing system size [7]. It is indeed true that the exact ground states of the even N samples have zero chirality for $N = 12, 18, 24, 36$. The present work indicates that this may not be true for the odd N samples.

4.3 Chern index of the low lying $S=1/2$ levels

Searching for a quantity that might provide further evidence for or against collective chiral behavior we followed the proposal of Haldane and Arovas [24] and computed the Chern number of the homogeneous ($\mathbf{k} = \mathbf{0}$) ground state of the $N = 9$ sample. It is a state which transforms as $R_{2\pi/3}\psi = e^{i2\pi/3}\psi$ under a spatial $2\pi/3$ rotation.

In this entire paragraph we shall concentrate on these specific states, which are the low lying homogeneous states of the $S = 1/2$ continuum that we have been discussing above. Anticipating further results we shall refer to these eigenstates as “chiral” states henceforth. In the $N = 27$ sample, 36 of the 39 homogeneous states of this low lying continuum are chiral states, and we expect that all states of this continuum will be chiral in the thermodynamic limit. The Chern number is a topological property of the eigenfunctions of a system constrained by twisted boundary conditions [24]. If \mathbf{T}_1 and \mathbf{T}_2 denote the two vectors defining the sample cell, the twisted boundary conditions are defined by

$$S^\pm(\mathbf{R}_i + \mathbf{T}_j) = e^{\pm i\Phi_j} S^\pm(\mathbf{R}_i), \quad (4)$$

where Φ_1 and Φ_2 are the two angles defining the rotations of the spins around the z -direction. Let us denote by $|n(\Phi_1, \Phi_2)\rangle$ the eigenstates obtained by adiabatic evolution of the state $|n\rangle$ under the twists. The Chern index of the ket $|n\rangle$ is defined by

$$C(n) = \frac{i}{2\pi} \int_0^{2\pi} d\Phi_1 \int_0^{2\pi} d\Phi_2 \varepsilon_{ab} \langle \frac{\partial n}{\partial \Phi_a} | \frac{\partial n}{\partial \Phi_b} \rangle. \quad (5)$$

It is a topological constant that can take only integer values. It can only be non-zero for complex, degenerate eigenstates, and, as will be discussed below, a non-zero value of the Chern number of an eigenstate $|n\rangle$ implies peculiar physical properties of the system in this state. A formula equivalent to (4) is obtained by the use of Stokes’ theorem:

$$C(n) = 2\pi i \oint \langle n | dn \rangle. \quad (6)$$

It shows that the Chern number counts the number and the nature of the singularities of the phase of $|n(\Phi_1, \Phi_2)\rangle$ in the (Φ_1, Φ_2) Brillouin zone. It is a measure of the vorticity of this phase. For computational purposes the expression (4) is quite cumbersome. A more convenient expression is obtained by changing to a spatially varying reference frame for the spin variables such that the twisted boundary conditions (3) are replaced by periodic ones. The main steps in this procedure are as follows: let \mathcal{S}_0 be the reference frame for the spin at the origin \mathbf{R}_0 of the sample. Then the reference frame at the site \mathbf{R}_i is obtained from \mathcal{S}_0 by a rotation through the angle

$$\Theta(\mathbf{R}_i) = (\mathbf{R}_i - \mathbf{R}_0)(\mathbf{e}_1\theta_1 + \mathbf{e}_2\theta_2), \quad (7)$$

around the z -direction, where $\mathbf{e}_1, \mathbf{e}_2$ span the Bravais lattice and θ_1, θ_2 are the increments of the rotation angle along \mathbf{e}_1 and \mathbf{e}_2 . θ_1, θ_2 are chosen such that they add up to the twist angles, *i.e.*:

$$\Phi_a = \Theta(\mathbf{R}_0 + \mathbf{T}_a), \quad a = 1, 2. \quad (8)$$

In this spatially varying reference frame the Hamiltonian reads

$$\tilde{\mathcal{H}}(\theta_1, \theta_2) = 2 \sum_{\langle i,j \rangle} \{ S_i^z S_j^z + 1/2 (e^{i\chi_{ij}} S_i^+ S_j^- + h.c.) \}, \quad (9)$$

where

$$\chi_{ij} = \Theta(\mathbf{R}_i) - \Theta(\mathbf{R}_j). \quad (10)$$

The Chern number in the eigenlevel $|n\rangle$ associated with the eigenvalue \tilde{E}_n of $\tilde{\mathcal{H}}$ is then readily obtained as the average over the (Φ_1, Φ_2) Brillouin zone of the function:

$$\mathcal{K}_{ab} = 2\pi\epsilon_{ab} \sum_{p \neq n} \frac{\langle n | \frac{\partial \tilde{\mathcal{H}}}{\partial \Phi_a} | p \rangle \langle p | \frac{\partial \tilde{\mathcal{H}}}{\partial \Phi_b} | n \rangle}{(\tilde{E}_n - \tilde{E}_p)^2}, \quad (11)$$

where

$$\frac{\partial \tilde{\mathcal{H}}}{\partial \Phi_a} = i \sum_{i,j} (e^{i\chi_{ij}} S_i^+ S_j^- - h.c.) \frac{\partial \chi_{ij}}{\partial \Phi_a}. \quad (12)$$

The phase angle χ_{ij} depends linearly on Φ_a through the local twists (7), and $\frac{\partial \chi_{ij}}{\partial \Phi_a}$ is a linear combination of the space components of $(\mathbf{R}_i - \mathbf{R}_j)$.

The computation of the Chern number is a heavy task. After a thorough study of the chiral ground state of the $N = 9$ sample we can ascertain that for this state the Chern number is +1 (respectively -1 for the complex conjugate eigenstate). In view of our study of the singular points of $\tilde{E}_n(\Phi_1, \Phi_2)$ in the chiral states of the $N = 21$ and 27 samples, we expect it to be odd for any of the low lying doublets and most probably equal to ± 1 for the ground state (in this last case, we find only one conical point at $(\Phi_1, \Phi_2) = (0, 0)$).

The physical significance of the Chern index may be inferred from the expression (11) which is a zero frequency Kubo response function. The general formula for the response of an observable B to an excitation of the observable A reads at $T = 0$:

$$\chi_{BA}^{T=0}(\omega = 0) = 2\hbar \text{Im} \sum_{p \neq n} \frac{\langle n | B | p \rangle \langle p | \dot{A} | n \rangle}{(\tilde{E}_n - \tilde{E}_p)^2}, \quad (13)$$

where \dot{A} is the time derivative of the observable coupled to the external field. Comparing this general form of the response function with the special form (11), one sees that in (11) both operators \dot{A} and B are linearly connected with the total spin currents $J_a = i \sum_{\langle i,j \rangle} (e^{i\chi_{ij}} S_i^+ S_j^- - h.c.)$, where the $\sum_{\langle i,j \rangle}$ is to be restricted for $a = 1, 2$ to the bonds along the directions \mathbf{e}_1 and \mathbf{e}_2 . The external field that drives a spin current along, *e.g.* the \mathbf{e}_1 -direction, is a magnetic field with a constant gradient in this same direction. This is most easily seen on the example of the square lattice, where the relation between (θ_1, θ_2) and (Φ_1, Φ_2) is diagonal. The case of the *kagomé* lattice is technically slightly more involved, but the physics is the same. The perturbation of the Hamiltonian $\tilde{\mathcal{H}}$ induced by a magnetic field B^z with a constant gradient in the \mathbf{e}_1 -direction is given by

$$V = - \sum_i \alpha_1 X_{i1} S_i^z, \quad (14)$$

where $X_{i1} = (\mathbf{R}_i \cdot \mathbf{e}_1)$. Here, α_1 is the linear increment of the magnetic field over one lattice constant. (Here and in the sequel we set \hbar , the Bohr magneton and the gyromagnetic factor equal to unity.) Identifying the observable A as $A = \sum (\mathbf{R}_i \cdot \mathbf{e}_1) S_i^z$ one sees immediately that its time derivative is the total spin current in the \mathbf{e}_1 -direction:

$$\dot{A} = -i[A, H] = \sum_{\langle i,j \rangle} \text{Im} \{ e^{i\chi_{ij}} S_i^+ S_j^- \} (X_{i1} - X_{j1}) = J_1. \quad (15)$$

In the orthogonal geometry one has the simple relation $J_a = \frac{\partial H}{\partial \Phi_a}$, $a = 1, 2$, and the expression (11) takes the form of a transverse current-current correlation function:

$$\mathcal{K}_{ab} = 2 \text{Im} \sum_{p \neq n} \frac{\langle n | J_a | p \rangle \langle p | J_b | n \rangle}{(\tilde{E}_n - \tilde{E}_p)^2}. \quad (16)$$

This correlation function measures the transverse current J_2 generated in the \mathbf{e}_2 -direction by a gradient of the magnetic field B^z along the \mathbf{e}_1 -direction. Hence we arrive at the linear relation

$$\langle J_2 \rangle = 2\pi \mathcal{K}_{ab} \frac{\partial B^z}{\partial X_1}. \quad (17)$$

At this point it is important to note that the above derivation of (17) is purely formal. It is a linear expansion with respect to the perturbation V , (14), which, as it is obvious from (14), becomes arbitrarily large in the thermodynamic limit. Thus, (17) cannot be considered a physically meaningful relation between the spin current and the gradient of the inhomogeneous magnetic field. However, by employing a gauge argument Haldane and Arovas [24] show that a physically sound relation is obtained from (17), if the response function \mathcal{K} is replaced by its average over the torus $0 < \Phi_1, \Phi_2 < 2\pi$ of the twist angles, *i.e.* by the Chern number $C(0)$.

From the mathematical point of view the Chern number of this spin system is equivalent to the TKNN index of the Quantum Hall Effect (QHE) [25], and as in the QHE the spin 1/2 excitations are separated from the $S = 0$ ground state by a finite gap (see $\Delta_{S=1/2}$ in Fig. 3). But there are very definite and crucial differences: i) In our spin system, contrary to the QHE, the parity and time reversal invariance are not externally broken. Eigenstates with positive and negative Chern number are degenerate. ii) In each spin sector there is a continuum of excitations adjacent to the ground state and there may be couplings between the two sectors under the effect of an external magnetic field. So it is difficult to imagine how the microscopic rigidity associated with the Chern number could become manifest on a macroscopic level. iii) Finally our numerical results seem to indicate that the creation of two spin 1/2 excitations is *less* favorable than the creation of a spin 1 excitation: $2E_{\Delta S=1/2} > E_{\Delta S=1}$ (see Fig. 3), but because of the uncertainties in the extrapolation procedure, this last result is not entirely reliable. The question of whether the excitation of two spin 1/2 entities is energetically more or less favorable than the creation of one

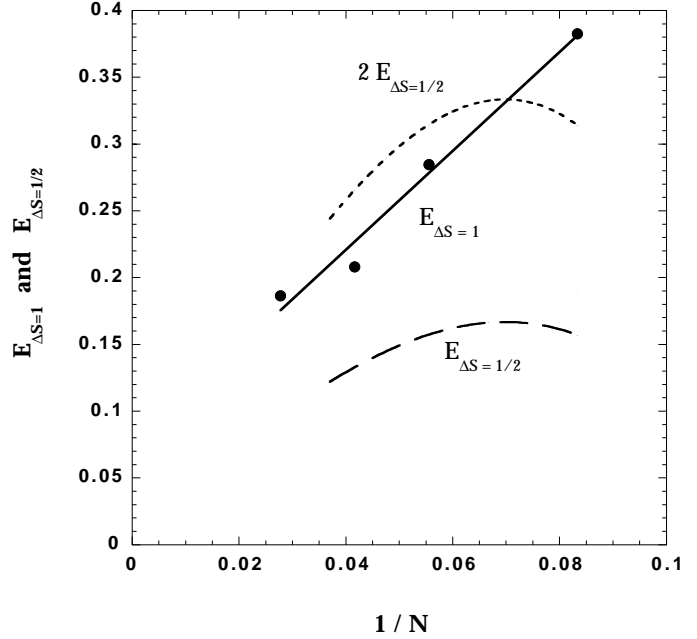


Fig. 3. $E_{\Delta S=1/2}$: energy for creating one $S = 1/2$ excitation. This energy is the difference between the interpolated ground state energies for $S = 1/2$ and $S = 0$ (see Fig. 1). $E_{\Delta S=1}$: energy for creating one $S = 1$ excitation in the even samples (magnetic gap). The graphs shown in this figure are based on polynomial interpolations between the numerical data. Continuations of these graphs to larger values of N ($N > 27$ for $E_{\Delta S=1/2}$ and $N > 36$ for $E_{\Delta S=1}$) are unwarranted.

$S = 1$ excitation is indeed an major open point. In the first case the spin 1/2 excitations would be true thermodynamic excitations, in the opposite case they could only appear as metastable excitations in sophisticated dynamic experiments. Whatever the answer to this last question may be, the KHA is certainly not a system which fits easily into the frame of standard continuum chiral theories. In any case the symmetries of these spin 1/2 excitations are features that will survive in the thermodynamic limit although the states may be metastable in this limit. We may hypothesize that their non-zero Chern number, since it is a quantum number, will also survive in the thermodynamic limit. In this picture the chiral $S = 1/2$ states of the KHA can certainly not be viewed as simple bound states of three spins 1/2 in a sea of singlets: a cluster of three spins in its $S = 1/2$ ground state has indeed a nonzero expectation value of the chirality $\Xi_{123} = \pm\sqrt{3}/2$, but its Chern index is zero. The Chern number is a measure of a topological rigidity of the N -particle states, which is by itself unique. To our knowledge the KHA is the first system with a Hamiltonian that does not break parity and time reversal invariance, where a non-zero Chern number has been observed.

5 Conclusion

In conclusion, our numerical study of the low lying spectra of the spin 1/2 KHA leads us to assert that this system is

a strange spin liquid: its lowest excitations are soft non-magnetic excitations whose existence in a spin system has not been anticipated in previous theoretical investigations of antiferromagnets. The absence of a gap between these excitations and the ground state may signal that the KHA is, at $T = 0$, in a critical state with arbitrarily long-ranged singlet-singlet correlations. In any case, the existence of this continuum of low energy nonmagnetic modes in the KHA already contradicts the conception of the ground state of this system as a short-range RVB state. It also contradicts the classical and semi-classical pictures of the KHA which are all more or less based on the idea of preferred planar spin arrangements, which imply a breaking of the $SU(2)$ symmetry. As has been shown in previous work [12,13], the spectra of *finite* samples of a spin system have to exhibit a certain signature, if a breaking of the $SU(2)$ symmetry is to occur in the *thermodynamic limit*. The spectra of the finite samples of the KHA do *not* show this signature [14], and thus a planar arrangement of the spins of the KHA can be ruled out.

In the sea of long-range correlated singlets an unpaired spin 1/2 is surrounded by transverse spin currents, which are another signature of long range correlations. The non zero Chern number of the $S = 1/2$ states of the KHA is a proof of some kind of microscopic rigidity (sensitivity to the boundary conditions), which is usually found in chiral theories in the literature. However, in standard chiral theories these spin 1/2 excitations are indeed the lowest excitations of the system, which is definitely not the case for the KHA. We do not know if these spin 1/2 excitations will be true thermodynamic excitations or only metastable ones, but they might show up in dynamical experiments (transverse spin diffusion, spin echoes).

To conclude we want to emphasize that there is some evidence that this kind of ground state and low lying excitations (critical singlets and gapped magnetic excitations) are robust properties of the KHA: finite perturbations (second neighbor interactions) are needed to drive the KHA into Néel ordered states [14,23], and the XY model on the *kagomé* lattice has a spectrum that is similar to the spectrum of the Heisenberg model. Thus, the above properties of the KHA with nearest neighbor interactions may be facets of a new state of frustrated quantum antiferromagnets which is generically different from the Néel ordered state on the one hand and from the short-range RVB state on the other [26–31]. The low energy physics of the KHA may be embodied in real compounds: in spite of the fact that SrCrGaO is a spin 3/2 compound with many complexities (dilute triangular planes of spins intercalated between the *kagomé* planes, role of the defects), some of its unusual properties (low temperature specific heat, dynamical spin correlations) may be explicable on the basis of the present results.

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